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The spread of a turbulent plane jet issuing into a parallel moving airstream

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The results of an experimental investigation into the development of a turbulent plane jet issuing into a parallel moving airstream are described. On the basis of a simple dimensional argument, it is shown that the results for the spread of jets with different ratios of jet nozzle to free-stream velocity can be collapsed into a single universal curve provided the effective origins of the various sets of data can be shifted. Evidence is found of a change in structure of the jet from a self-preserving plane jet flow near the origin of the flow towards a self-preserving wake type of flow far downstream from the origin. This change of structure is compared with a prediction based on a simple application of Townsend's large-eddy hypothesis. It is shown that the hypothesis does not account for the way in which the jet structure changes and possible reasons for this are briefly discussed. Finally, some comments are made on the usefulness of the various theories of jet spreading.

1. Introduction

Although the spread of turbulent jets issuing into parallel moving airstreams has now been the subject of a number of theoretical treatments (Squire & Trouncer 1944; Abramovich 1958; Hill 1965), reliable experimental data on these flows are still comparatively sparse. The present paper contains results of an experimental investigation into the spread of a plane jet in a moving airstream and it is the intention of the paper to draw attention to certain features of the flow which are normally only briefly considered. Of particular interest is the variation of eddy Reynolds number that occurs in this flow and its relationship to Townsend's large-eddy hypothesis.

A basic assumption which is invariably implicit in any work that is carried out on fully turbulent free jet and wake flows is that some way downstream from the origin of the flow, the flow becomes independent of the precise conditions at the origin. For example, in the case of a wake flow, the flow is assumed to depend only on the overall drag of the body producing the wake and not on its precise shape and size. In the case of a plane jet in a moving airstream, it is

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assumed that only the overall excess momentum flux of the jet is important and not details of the jet exit velocity and nozzle width as separate parameters. This assumption leads to a simple dimensional argument outlined in § 2 which suggests that the experimental results for the spread of a jet in a moving airstream with different values of jet nozzle to free-stream velocity can be collapsed into a single universal curve. In § 3.1, the experimental results are examined in the light of this simple argument and it is shown that the results can indeed be collapsed in this way provided an effective shift in the apparent origins of the various sets of data is allowed for. Some turbulence measurements are also discussed which further illustrate both the usefulness and limitations of the simple dimensional argument.

Another important feature of the spread of a plane jet in a moving airstream is the fact that it can exhibit a self-preserving structure in two limited regions of the flow. First, a self-preserving flow is possible when the velocity on the jet centre-line is much greater than the free-stream velocity. This flow spreads in such a way that the jet width δ is proportional to the distance x downstream from the origin of the flow and in which the excess of velocity U_0 on the jet centre-line over the free-stream velocity is proportional to $x^{-\frac{1}{2}}$. This flow, which will be referred to as the self-preserving 'pure' jet type of flow, has been the subject of a number of investigations usually in the limiting case of a jet issuing into still air. The second region in which a self-preserving flow is possible is far downstream of the flow origin when the jet centre-line velocity is approaching the free-stream velocity. This type of flow will be similar to the self-preserving wake flow investigated extensively by Townsend (1956) and in which $\delta \propto x^{\frac{1}{2}}$ and $U_0 \propto x^{-\frac{1}{2}}$. The essential difference between these two self-preserving flows as far as their general development is concerned is that the eddy Reynolds number for the 'pure' jet flow is about twice the wake flow value. Thus, in examining the structure of a plane jet in a moving airstream, one would expect to find that this was similar to the self-preserving 'pure' jet flow near the origin of the flow, but that it approached finally that of a self-preserving wake type of flow far downstream. The experimental results appear to fit in with these expectations as will be discussed in § 3.2, but the main point of interest is the way in which the eddy Reynolds number varies as the flow changes over from the one type of self-preserving flow to the other. The problem of predicting such a variation is a central one in turbulent shear flows, as it invariably arises in all non-self-preserving flows. The only really plausible explanation that has been put forward to account for it is due to Townsend (1956) in his large-eddy hypothesis and, in § 3.2, the experimental variation of eddy Reynolds number is compared with a straightforward application of this hypothesis. It is found that the observed variation of eddy Reynolds number occurs much more slowly than one would expect from the hypothesis and possible reasons for this are briefly discussed. Finally, in § 4, the inadequacy of existing theories for predicting the development of jets in moving airstreams is discussed.

2. Dimensional arguments and self-preserving flows

The flow to be considered is of a plane jet issuing into a parallel moving airstream of velocity U_1 (see figure 1). In the fully turbulent region some way downstream of the jet nozzle, it is assumed that the flow does not depend directly on

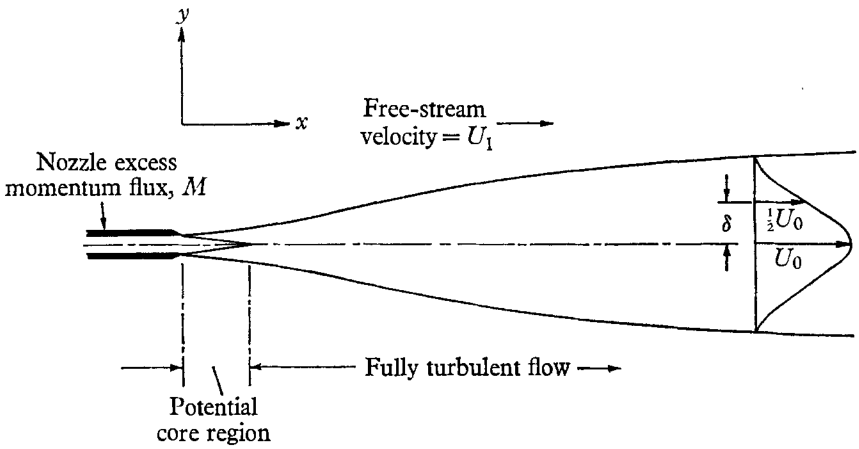

FIGURE 1. Schematic representation of a plane jet spreading into a moving airstream.

the precise nozzle conditions such as the jet exit velocity U_J and nozzle width h, but only on the excess momentum flux M defined by the momentum integral equation†

$$M(\text{a constant}) = \rho \int_{-\infty}^{\infty} U(U-U_1)\,dy. \tag{1}$$

Dimensional analysis then gives for this region of the flow

$$U_0/U_1 = F\{(x-x_0)/\theta\}, \tag{2a}$$

and

$$\delta/\theta = G\{(x-x_0)/\theta\}, \tag{2b}$$

where $\theta = M/\rho U_1^2$ is the momentum thickness of the jet. U_0 is the excess of velocity on the jet centre-line over the free-stream velocity U_1. δ is the y-ordinate at which $U = U_1 + \frac{1}{2}U_0$. x_0 is an apparent shift in the origin of the flow due to the particular nozzle conditions and is the only influence on the flow that the nozzle conditions are assumed to have. F and G are functions of $(x-x_0)/\theta$ only.

At this stage, the form of the functions F and G cannot be defined everywhere. However, they will approach the limiting forms appropriate to a self-preserving 'pure' jet type of flow as $(x-x_0)/\theta \to 0$ on the one hand and a self-preserving wake type of flow as $(x-x_0)/\theta \to \infty$ on the other. For future use in § 3 these limiting forms can be obtained conveniently from a simple theory using an eddy viscosity

† The turbulent terms in the momentum integral equation have been ignored because they contribute only about 3 % to the value of the momentum flux for a jet in still air and their contribution in the case of a jet in a moving airstream is even smaller.

coefficient in conjunction with the momentum and energy integral equations. This analysis gives for:

(*a*) the self-preserving 'pure' jet flow

$$\left(\frac{U_1}{U_0}\right)^2 = \frac{4I_2I'}{R_{T_J}I_3}\frac{x-x_0}{\theta}, \quad \frac{\delta}{\theta} = \frac{4I'}{R_{T_J}I_3}\frac{x-x_0}{\theta}; \tag{3a, b}$$

(*b*) the self-preserving wake type of flow

$$\left(\frac{U_1}{U_0}\right)^2 = \frac{4I_1I'}{R_{T_W}I_2}\frac{x-x_0}{\theta}, \quad \frac{\delta}{\theta} = 2\sqrt{\left(\frac{I'}{R_{T_W}I_2I_1}\right)}\sqrt{\left(\frac{x-x_0}{\theta}\right)}: \tag{4a, b}$$

where R_T is an eddy Reynolds number defined by

$$\frac{1}{R_T} = \frac{1}{I'}\int_{-\infty}^{\infty}\frac{\tau}{\rho U_0^2}\frac{df}{d\eta}d\eta, \tag{5}$$

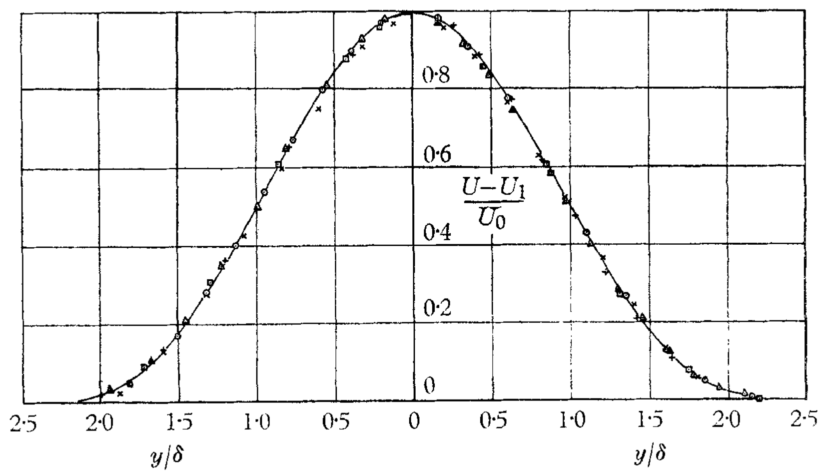

FIGURE 2. Mean velocity profiles of a plane jet in a moving airstream. $\times$, $U_0/U_1 = 6{\cdot}64$; $\triangle$, $U_0/U_1 = 1{\cdot}93$; $\odot$, $U_0/U_1 = 1{\cdot}13$; $+$, $U_0/U_1 = 0{\cdot}41$; $\square$, $U_0/U_1 = 20{\cdot}20$.

where τ is the shear stress and the suffices J and W refer to the 'pure' jet and wake type of flows respectively.

$$I_n = \int_{-\infty}^{\infty} f^n d\eta \; (n = 1, 2, 3) \quad \text{and} \quad I' = \int_{-\infty}^{\infty} (df/d\eta)^2 d\eta,$$

where $f(\eta)$ is the self-preserving mean velocity profile for the two flows defined by

$$U = U_1 + U_0 f(\eta), \tag{6}$$

where $\eta = y/\delta$. The function $f(\eta)$ for the jet and the wake flow need not be the same of course but, as shown by some experimental results in figure 2, mean velocity profiles in a plane jet in a moving airstream seem to be geometrically similar throughout the entire flow and any differences that there may be between profiles for large and small values of U_0/U_1 are too small to be significant.

It should be mentioned here that the use of the integral equations to obtain limiting forms for the functions F and G is preferred to the more usual classical eddy viscosity and mixing-length solutions found in standard text-books because the experimental mean velocity profile function can be used. This will obviously lead to better agreement with the experimental results and also to values of the eddy Reynolds number which are more meaningful in terms of representing the integral property of the shear stress as expressed by (5).

For the mean-velocity-profile function shown in figure 2, the following values of I_n and I' have been obtained:

$$I_1 = 2\cdot025; \quad I_2 = 1\cdot467; \quad I_3 = 1\cdot197; \quad I' = 1\cdot0914.$$

Results of Bradbury (1965) and Townsend (1956) for the self-preserving 'pure' jet flow and the self-preserving wake flow respectively give values of $R_{T_J} = 33\cdot4$ and $R_{T_W} = 14\cdot7$, which, upon substitution into (3) and (4), give:

(*a*) for the self-preserving 'pure' jet

$$\left(\frac{U_1}{U_0}\right)^2 = 0\cdot16\frac{x-x_0}{\theta}, \quad \frac{\delta}{\theta} = 0\cdot109\frac{x-x_0}{\theta}; \tag{7a, b}$$

(*b*) for the self-preserving wake type of flow

$$\left(\frac{U_1}{U_0}\right)^2 = 0\cdot41\frac{x-x_0}{\theta}, \quad \frac{\delta}{\theta} = 0\cdot316\sqrt{\left(\frac{x-x_0}{\theta}\right)}. \tag{8a, b}$$

These expressions will be used later in § 3.2.

3. Discussion of experimental results

The model used in the experiments comprised a wing of 27 in. chord and approximately 1 in. thick, which spanned the width of the 4 ft. × 3 ft. closed return tunnel at Queen Mary College. The plane jet exhausted from the centre 18 in. of the wing trailing edge and, to ensure that the jet flow was closely two-dimensional, two plywood false walls were mounted across the tunnel, one on either side of the jet span. Details of the air supply to the model are described by Bradbury (1963, 1965). A traversing gear enabled measuring instruments to be traversed both longitudinally along the axis of the jet and laterally across the jet and it was possible to make measurements up to about 25 in. downstream from the jet nozzle. Two jet widths of 0·375 and 0·125 in. were used in the experiments. The former jet width was used primarily to study the flow at the larger values of U_0/U_1 and the smaller jet to study the flow at small values of U_0/U_1.

The measurements consisted primarily of mean velocity traverses with Pitot and static tubes to determine the spread of the jet width and the decay of the centre-line velocity. However, a few turbulence measurements with a DISA constant-temperature hot-wire anemometer were also made. These were measurements of $\overline{u^2}/U_0^2$ along the centre-line of the jet over a wide range of values of U_0/U_1 and also some measurements of the shear stress distribution across the jet.

In order to reduce the data to the form of (2*a*) and (2*b*), the value of the excess momentum flux M had to be known. This could not be obtained generally from the ratio of the jet exit velocity to free-stream velocity because, first, a significant contribution to the net momentum flux came from the drag of the wing for the lowest values of U_J/U_1 tested and, secondly, for the 0·125 in. nozzle, the velocity distribution across the nozzle was not uniform. The momentum flux was therefore calculated from the mean-velocity traverses across the jet in the fully turbulent flow downstream of the nozzle. In the earlier tests, about ten lateral traverses would be made in this region for a given value of U_J/U_1 and it was found that the values of the excess momentum flux calculated from these traverses were within about 5 % of each other, which gives a guide to the general accuracy of the experiments. No three-dimensional effects of the sort discussed by Gartshore (1965) were found. For the reduction of the data, the average value of the excess momentum flux obtained from the lateral traverses was used. Once the general accuracy of the experiments had been established, further tests were made in which only three lateral traverses were made for each value of U_J/U_1 tested. These traverses enabled the excess momentum flux to be obtained and, since the mean-velocity profiles appear to be geometrically similar, a further single traverse along the axis of the jet measuring U_0 was sufficient to provide complete information about the development of the mean velocity field.

3.1. *The effective origin shifts and independence of nozzle conditions*

In the dimensional arguments of § 2, the precise nozzle conditions are assumed not to influence the structure and development of the fully turbulent jet some way downstream from the nozzle. However, the nozzle conditions do influence the distance necessary to establish this independence, as is shown by some results in figure 3 of the variation of $(U_1/U_0)^2$ with x/θ for various values of jet exit to free-stream velocity. However, it is clear that, if an effective shift in the origins of the various sets of data is allowed for, a single universal curve in accordance with (2*a*) can be obtained provided the results nearest the nozzle for each velocity ratio U_J/U_1 are excluded. These latter results are clearly directly influenced by the precise nozzle conditions. The complete results with the relevant origin shifts are shown in figures 4 and 5 for both the decay of the centre-line velocity and the spread of the jet width. Figure 4 shows results for the larger values of U_0/U_1, where similarities to the self-preserving 'pure' jet type of flow would be expected. Figure 5 gives results for smaller values of U_0/U_1, where an approach to the self-preserving wake type of structure would be expected.

The shifts in the effective origins of the jet flow necessary to produce the universal curves are of no special significance because they are certainly dependent on the particular model used in the experiments. Nevertheless, it is perhaps significant that these shifts are too large to ignore and, therefore, in practical problems, it is necessary to be able to predict them. Unfortunately, this is not likely to be very easy.

Confirmation of the universality of the results of figures 4 and 5 to represent the spread of a plane jet in a moving airstream can be obtained from the turbulence measurements of $\overline{u^2}/U_0^2$ on the centre-line of the jet. These were made at a number of fixed longitudinal stations over a range of values of U_0/U_1 and the results are shown in figure 6. If the flow is independent of the precise nozzle conditions, $\overline{u^2}/U_0^2$ on the jet centre-line will be a function of U_0/U_1 only. This is clearly so for the results shown in figure 6 up to some limiting value of U_0/U_1 which is dependent on the particular longitudinal station. Beyond this limiting

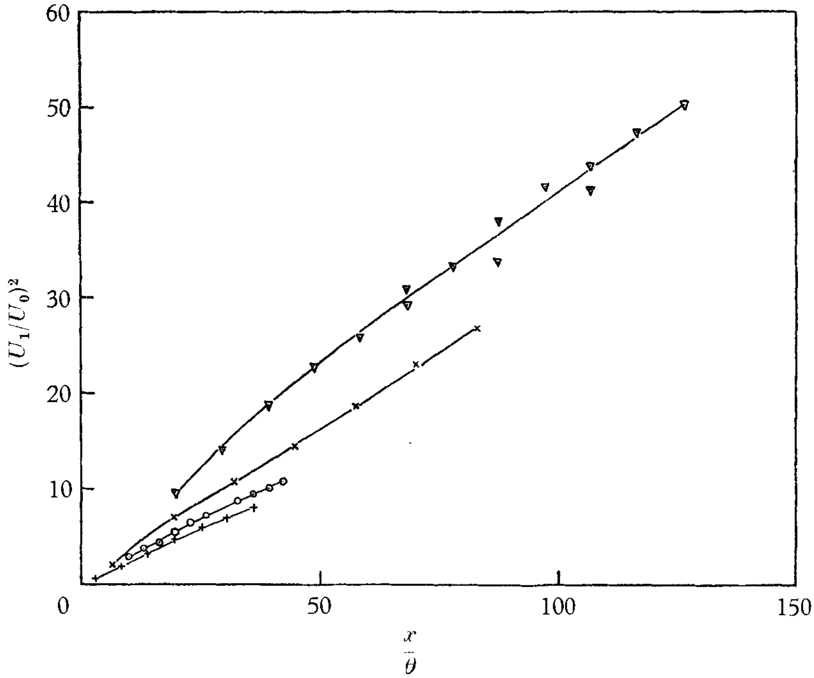

FIGURE 3. Decay of centre-line velocity for various ratios of jet nozzle to free-stream velocity ($\frac{1}{8}$ in. nozzle). U_1/U_J: ∇, 0·47; $\times$, 0·4; $\odot$, 0·36; $+$, 0·23.

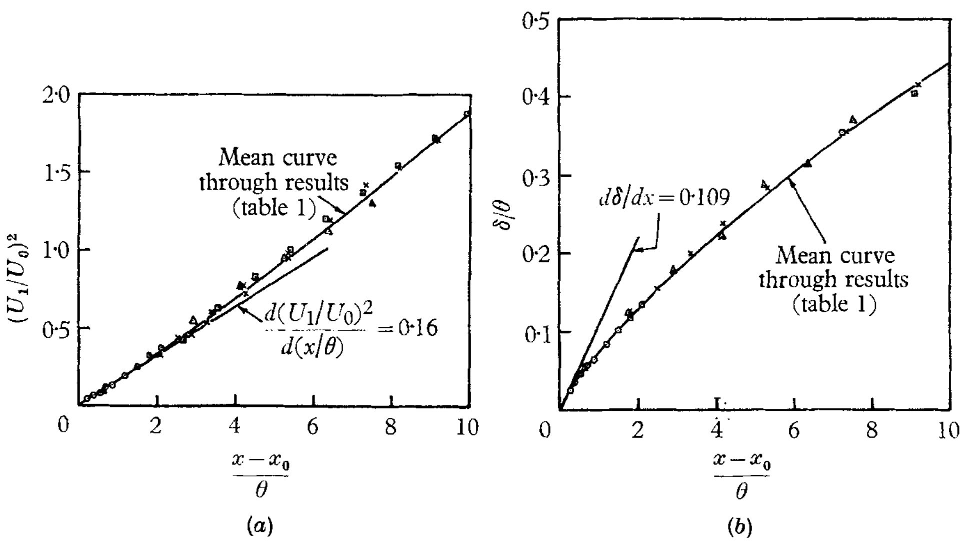

FIGURE 4. The development of a plane jet $(x-x_0)/\theta < 10$. (a) the centre-line velocity. (b) The jet width.

	U_1/U_J			U_1/U_J	
$\odot$	0·16	$\frac{3}{8}$ in. nozzle	$\triangle$	0·16	$\frac{1}{8}$ in. nozzle
$\times$	0·3		$\square$	0·23	

value, the effects of the nozzle conditions begin to influence the flow and the values of $\overline{u^2}/U_0^2$ deviate from the 'universal' values.

An important feature of the turbulence measurements shown in figure 6 is that they show that with the present experimental set-up where measurements beyond $x/h = 250$ were not possible, a jet flow independent of the precise nozzle conditions could only be obtained for $(U_1/U_0)^2 \leqslant 25$. Thus, mean velocity results such as those shown in figure 3 obtained for values of $(U_1/U_0)^2 \geqslant 25$ must be omitted from the 'universal' curves of figure 5.

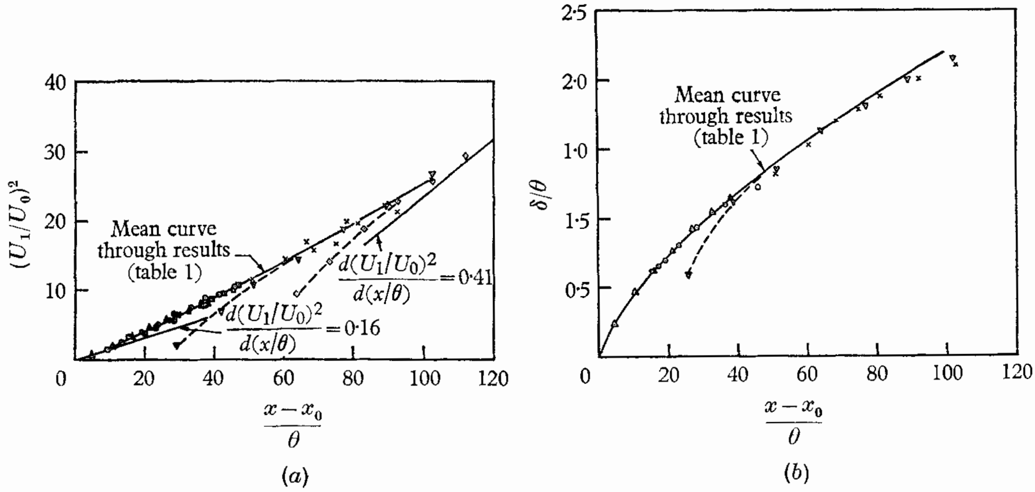

FIGURE 5. The development of a plane jet $(x-x_0)/\theta \leqslant 120$. (*a*) The centre-line velocity (*b*) The jet width.

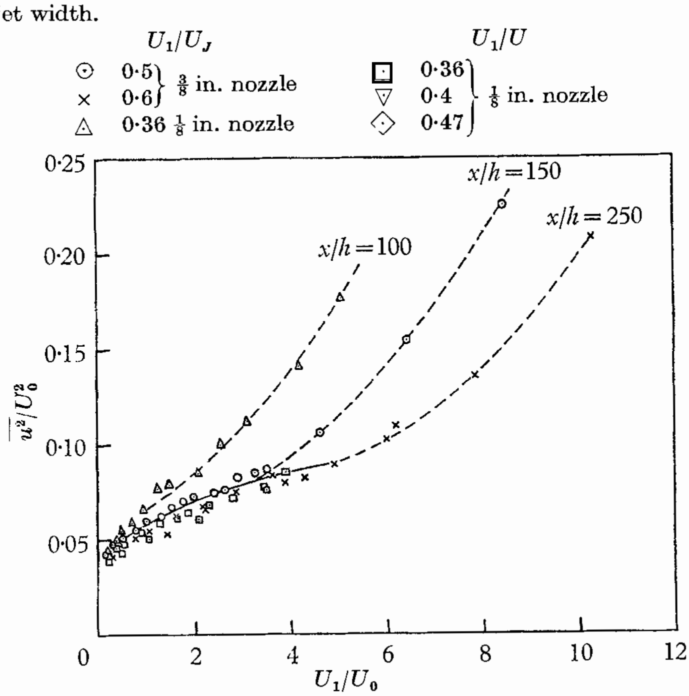

FIGURE 6. $\overline{u^2}/U_0^2$ on the jet centre-line ($\frac{1}{8}$ in. nozzle). △, $x/h = 100$; ⊙, $x/h = 150$; ⊡, $x/h = 200$; ×, $x/h = 250$.

Because of the scarcity of jet-spreading data, it seems worth while to tabulate the present results for the 'universal' jet flow so that they are readily available for any further work that may be carried out on the subject. Thus a table giving details of the development of the jet is presented in the appendix 1.

3.2. *The eddy Reynolds number variation*

Reference to the mean-velocity results in figures 4 and 5 shows that as the jet spreads downstream, the value of $d(U_1/U_0)^2/d(x/\theta)$ increases above the 'pure' jet flow value of 0·16 and seems to be approaching the wake flow value of 0·41 at the limit at which experiments were carried out. This suggests that the expected change in structure, as discussed in § 1, is occurring and this is confirmed by the turbulence measurements in figure 6 which show that $\overline{u^2}/U_0^2$ on the jet centre-line increases as U_0/U_1 decreases. In fact, the values of $\overline{u^2}/U_0^2$ at the smallest values of U_0/U_1 appear somewhat larger than Townsend's wake flow values but, in view of the inevitable uncertainties in hot-wire measurements, this is probably not significant.

The eddy Reynolds number variation as the jet spreads downstream can conveniently be calculated from the energy and momentum integral equations, namely

$$\tfrac{1}{2}\rho \frac{d}{dx}\int_{-\infty}^{\infty} U(U^2-U_1^2)\,dy = -\int_{-\infty}^{\infty} \tau\frac{\partial U}{\partial y}\,dy, \tag{9}$$

and

$$\int_{-\infty}^{\infty} U(U-U_1)\,dy = \frac{M}{\rho}, \tag{10}$$

which upon substitution of the mean-velocity-profile function (6) and the eddy Reynolds number expression (5) give

$$(3\lambda^2 I_3 + 6\lambda I_2 + 2I_1)\frac{\delta}{\lambda^3}\frac{d\lambda}{dx} + (\lambda^2 I_3 + 3\lambda I_2 + 2I_1)\frac{1}{\lambda^2}\frac{d\delta}{dx} = -\frac{2I'}{R_T}, \tag{11}$$

$$\delta(\lambda^2 I_2 + \lambda I_1) = M/\rho U_1^2, \tag{12}$$

where $\lambda = U_0/U_1$.

Thus, from experimental values of U_0/U_1 against x/θ, the variation of eddy Reynolds number can be calculated. The results of these calculations are shown in figure 7 along with a few values obtained from direct measurements of shear stress with a hot-wire anemometer. The values obtained from the hot-wire measurements are very sensitive to errors in the hot-wire measurements and, therefore, the agreement between the two sets of values of $1/R_T$ can be regarded as satisfactory and confirms that the calculated values were not being affected in a gross way by any spurious three-dimensional effects.

The results shown in figure 7 show that $1/R_T$ increases with increasing distance downstream although it is still some way from the wake flow value at the limit at which experiments were carried out. Nevertheless, it is interesting that this variation has been found because, in the case of an axisymmetric jet, Maczynski (1962) found no tendency towards a wake-flow type of structure even though his experiments extended to much smaller values of U_0/U_1 than was possible in the present tests.

The only really plausible explanation that has been put forward to account for the variation of eddy Reynolds number is due to Townsend (1956) in his large-eddy hypothesis. Townsend used this hypothesis originally to explain the differences in the eddy Reynolds numbers found in the various self-preserving

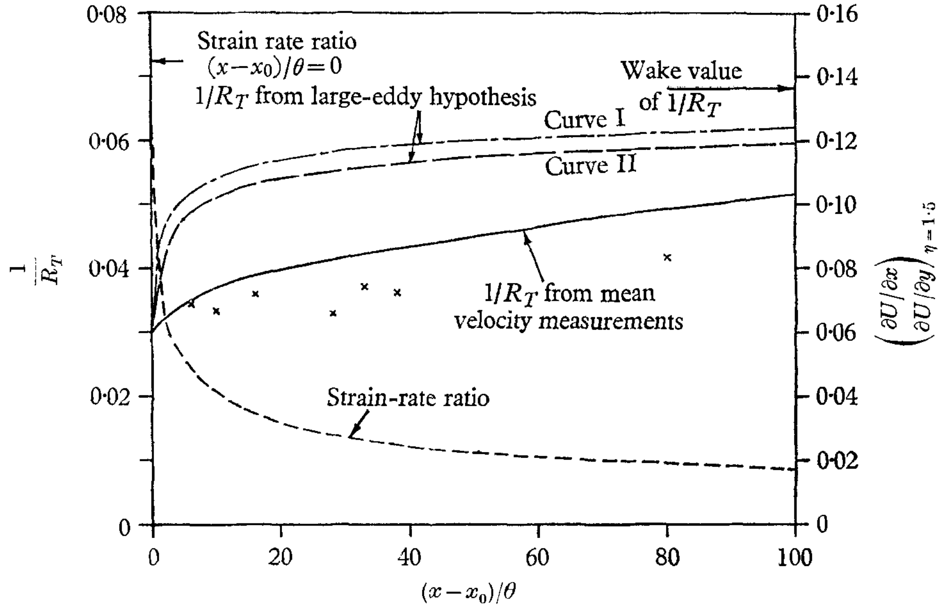

FIGURE 7. Variation of strain-rate ratio and eddy Reynolds number in the jet. × – $(1/R_T)$ from hot-wire measurements.

flows and he effectively suggested the following expression for the ratio of the eddy Reynolds number in a self-preserving 'pure' jet to the wake flow value, namely

$$\frac{R_{T_J}}{R_{T_W}} = \exp\left[-\beta\left(\frac{\partial U/\partial x}{\partial U/\partial y}\right)_t\right], \tag{13}$$

where $[(\partial U/\partial x)/(\partial U/\partial y)]_t$ is the strain-rate ratio at some typical station in the outer region of the jet and β is a constant with a value of about 6 according to Townsend. The strain-rate ratio in the wake flow is zero. Now, using (6) gives

$$\frac{\partial U/\partial x}{\partial U/\partial y} = \frac{\delta}{U_0}\frac{dU_0}{\partial x}\left(\frac{f}{f'}\right) - \frac{d\delta}{dx}\eta. \tag{14}$$

If this strain-rate ratio is evaluated at $\eta = 1\cdot5$ as a typical station, then, for the mean-velocity-profile function shown in figure 2, $f/f' = -0\cdot345$. Furthermore, in a self-preserving 'pure' jet, it is easy to show that $-(\delta/U_0)(dU_0/dx) = \frac{1}{2}d\delta/dx$, so that using values of $d\delta/dx = 0\cdot109$ and $R_{T_J} = 33\cdot4$ obtained by Bradbury (1963) for the 'pure' jet, and $R_{T_W} = 14\cdot7$ from Townsend's wake measurements, (13) gives $\beta = 5\cdot63$, which agrees well with Townsend's suggested value and thus apparently confirms the validity of the hypothesis. It is interesting now to apply (13) to the jet in a moving airstream. Unlike the self-preserving flows, the strain-rate ratio in this case is continually varying as the flow spreads downstream but we shall nevertheless apply (13), using local values of the strain-rate ratio to predict local values of eddy Reynolds number. For this to be a valid procedure, it is strictly necessary that the change in the strain-rate ratio during the life of a large eddy should be small. This assumption will be examined later.

The variation of the strain-rate ratio has been computed from the experimental results using (14) and is shown in figure 7 and is also tabulated in the appendix. The corresponding values of the eddy Reynolds number obtained from (13) are also shown in figure 7 as curve I. It is quite obvious that this rather straightforward application of the large-eddy hypothesis gives a much more rapid approach to the wake-flow type of conditions than is found in practice.

As a possible reason for the failure of the large-eddy hypothesis, the effect of the changing strain-rate ratio has been examined. It was found that during the lifetime of an eddy—typically $\beta/(\partial U/\partial y)$—the strain-rate ratio reduced roughly by a factor of two. An allowance for this effect was made by using the values of eddy Reynolds number calculated from (13) (i.e. curve I in figure 7) but shifting them a distance downstream of $\beta(U_1+U_0)/(\partial U/\partial y)$. The results of this very rough calculation are shown as curve II in figure 7. This shows that this possibility is also not an adequate explanation for the failure of the large-eddy hypothesis and other explanations must still be sought. Two possibilities arise here. The first is simply that the large-eddy hypothesis is basically incorrect. This seems unlikely because there is a good deal of evidence, both theoretical and experimental, to show that mechanisms at least similar to those postulated by Townsend are at work in self-preserving turbulent shear flows. The other possibility is that the hypothesis cannot be carried over to a non-self-preserving flow in the way that it has been done here. This seems a more likely explanation because, in addition to the effect of changing strain-rate ratio, there are also questions of the time necessary to establish large-eddy equilibrium and, as is well known, the time scales for production and dissipation tend to be somewhat longer than the lifetime of an eddy. Thus, it may be that the energy equilibrium as set out in the hypothesis is never established.

4. Comparison with theories of jet spreading

It is not the intention here to go into details about the various theories of jet spreading but it seems worth while illustrating the discrepancies between these theories and experiments. All the theories of jet spreading are essentially of the integral-equation type and they all assume that the mean-velocity profiles are geometrically similar. They can then be divided into two groups. First, there are those which effectively assume that the eddy Reynolds number is constant throughout the flow with a value appropriate to the self-preserving 'pure' jet flow. These theories clearly have the wrong asymptotic behaviour as $U_0/U_1 \to 0$. Theories of this sort have been developed by Squire & Trouncer (1944), Hill (1965) and Abramovich (1958). The second group of theories endeavours to allow for changes in the eddy Reynolds number by applying Townsend's large-eddy hypothesis in the form of (13) using local values of the strain-rate ratio to compute local values of the eddy Reynolds number.

These theories appear to have the correct asymptotic behaviour both as $U_0/U_1 \to \infty$ and also as $U_0/U_1 \to 0$, and will be referred to as variable eddy Reynolds number theories. Theories of this sort have been put forward by Bradbury (1963) and Gartshore (1965).

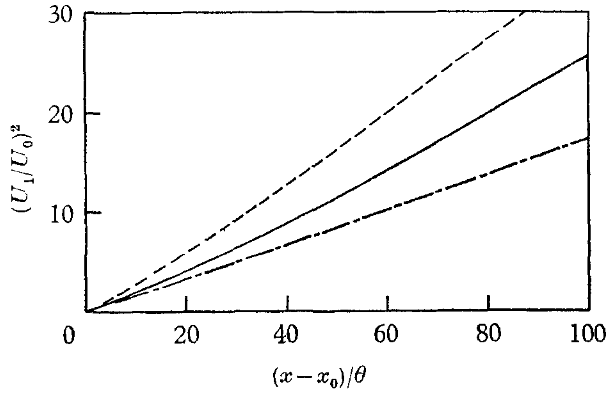

FIGURE 8. Comparison between theories and experimental results for jet centre-line velocity. ———, mean curve through experimental results (table 1, appendix); ----, variable eddy Reynolds number theory; -—-—, constant eddy Reynolds number theory.

To represent these two groups of theories, we can conveniently use the momentum and energy integral equations along with the experimental mean-velocity profile and, first, assume that R_T is a constant with a value of 33·4 (i.e. the 'pure' jet value) and, secondly, use (13) to give a variable eddy Reynolds number. There is no intrinsic value in the details of this analysis, which is given in full by Bradbury (1963), and it is sufficient here simply to compare the predictions of these theories with the experimental results. As might be expected from earlier arguments, neither of these two theories gives very good agreement with experiment (see figure 8) and, in fact, they are in error by almost equal amounts but in opposite directions.

5. Conclusions

The development of a plane jet in a parallel moving airstream has been studied. It has been shown that, provided an effective shift in the origin of the flow is allowed for, the flow some distance downstream of the nozzle is dependent only on the overall excess momentum flux of the jet and is independent of the precise nozzle conditions. Some evidence is found of a change in structure as the jet spreads downstream from a 'pure' jet type of flow near the nozzle towards a self-preserving wake type of structure far downstream, but it is clear that this change occurs at a much slower rate than might be suggested from a rather straightforward application of Townsend's large-eddy hypothesis. This finding is in accord with some somewhat similar results of Maczynski (1962) for a circular jet and shows that simple integral theories using Townsend's large-eddy hypothesis are not tenable. There seems to be some discrepancy here with the findings of Gartshore (1965), who found good agreement between such a theory and experimental results for the spread of a non-self-preserving wall jet flow.

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Appendix

Table 1 below lists the important features of a plane jet issuing into a parallel moving airstream. The values of the eddy Reynolds number R_T and the strain-rate ratio have been calculated from the data in the other columns using (11), (12) and (14).

$\frac{x-x_0}{\theta}$	$\frac{\delta}{\theta}$	$\left(\frac{U_1}{U_0}\right)^2$	$\frac{d(U_1/U_0)^2}{d(x/\theta)}$	R_T	$\left(\frac{\partial U/\partial x}{\partial U/\partial y}\right)_{\eta=1\cdot5}$
0	0	0	0·16	33·4	0·145
2	0·127	0·335	0·175	31·25	0·0659
4	0·221	0·696	0·186	29·7	0·0546
6	0·302	1·077	0·195	28·6	0·0484
8	0·375	1·474	0·202	27·7	0·0441
10	0·444	1·885	0·209	26·9	0·0412
12	0·508	2·306	0·212	26·6	0·0383
20	0·732	4·060	0·226	25·25	0·0316
30	0·970	6·390	0·240	24·0	0·0271
40	1·181	8·845	0·251	23·0	0·0243
50	1·374	11·41	0·262	22·1	0·0224
60	1·553	14·08	0·272	21·4	0·0210
70	1·723	16·85	0·281	20·75	0·0198
80	1·884	19·7	0·290	20·1	0·0189
90	2·039	22·63	0·296	19·7	0·0180
100	2·187	25·62	0·306	19·4	0·0173

TABLE 1

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